IDC452: Seminar Delivery

Simulating Plasma Physics

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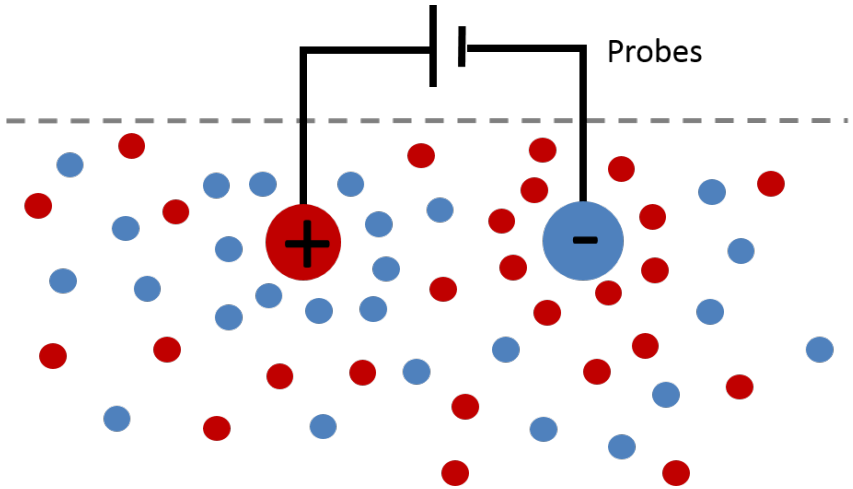
Saturday 26th March, 2022

Introduction

- Quasi-neutral gas exhibiting collective behaviour



- Some characteristic properties
  - Debye length; λ_D
  - Electronic thermal velocity; v_{th}
  - Plasma frequency; ω_{pe}

Governing equations (Electrostatic)

Purely Particle-based implementation:

$$\begin{aligned}\vec{F} &= \vec{F_e} + \vec{F_b} \\ &= q \cdot \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{|\vec{r} - \vec{r_i}|^3} \cdot (\vec{r} - \vec{r_i}) + q \cdot (\vec{v} \times \vec{B}_{ext})\end{aligned}$$

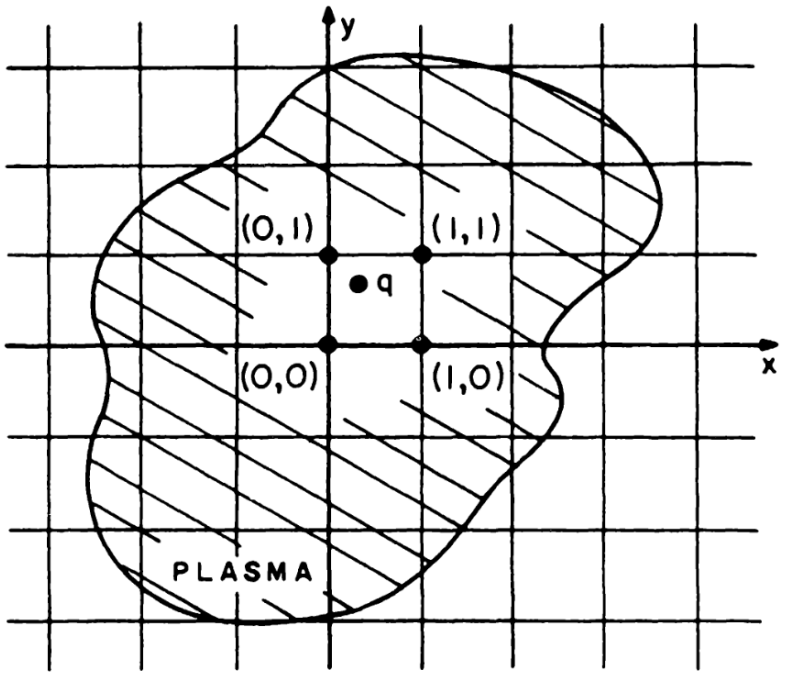
Particle + Field implementation:

$$\vec{F} = q \cdot (\vec{E} + \vec{v} \times \vec{B}_{ext})$$

$$\vec{E} = -\nabla\phi$$

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0}$$

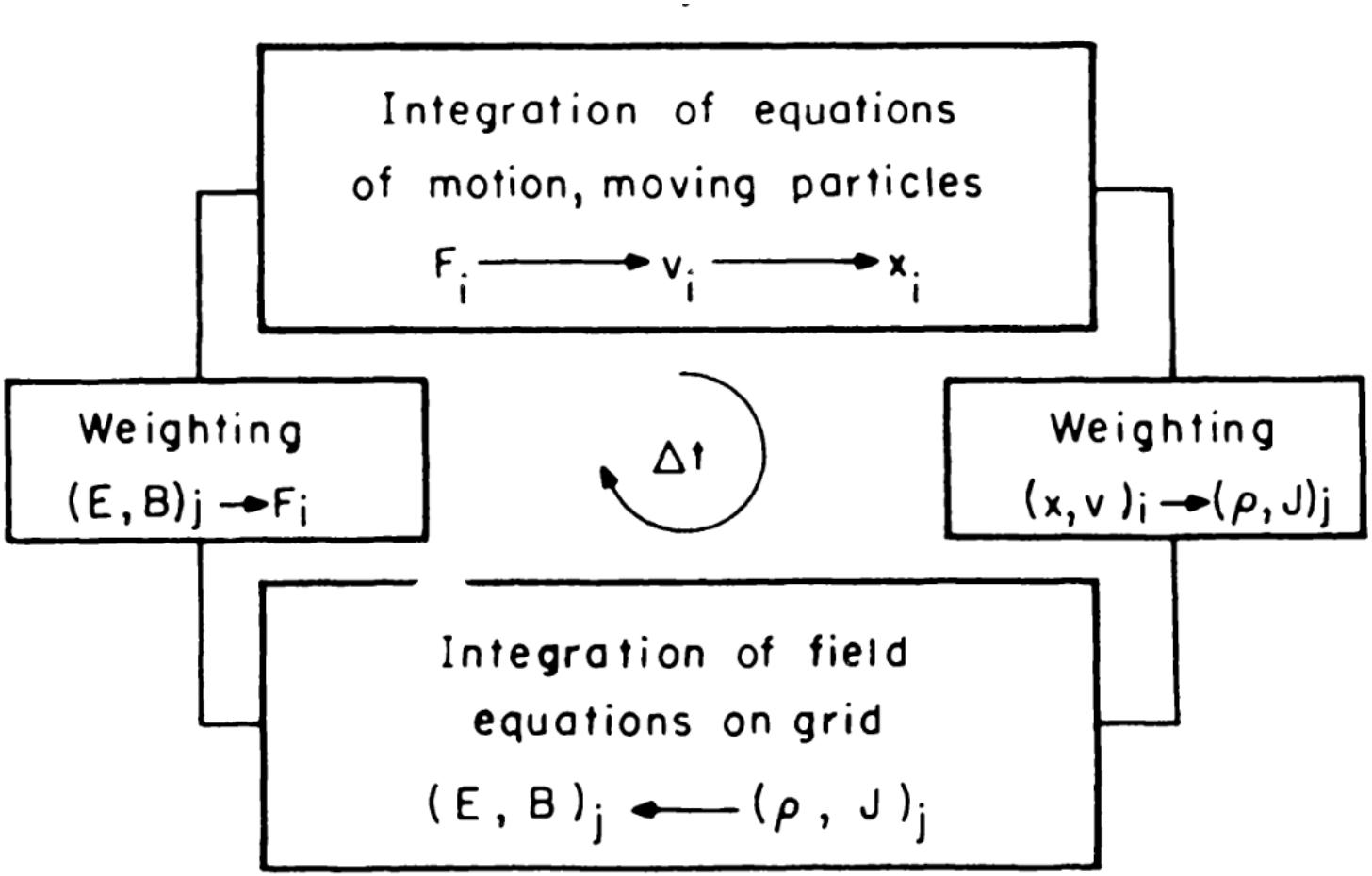
Particles and 'Super-Particles'



- Particles in continuous space
- Fields (ρ, ϕ, $\vec{E}$) on discrete grid; Δx
- Discrete time grid; Δt

$$\text{Particle} \underset{\text{move}}{\overset{\text{generate}}{\rightleftharpoons}} \text{Field}$$

Cyclic algorithm to update the system



Details - Particle evolution

We use a modified leapfrog integrator (Boris algorithm).

$$\boxed{\frac{x_{k+1} - x_k}{\Delta t} = v_{k+1}}$$

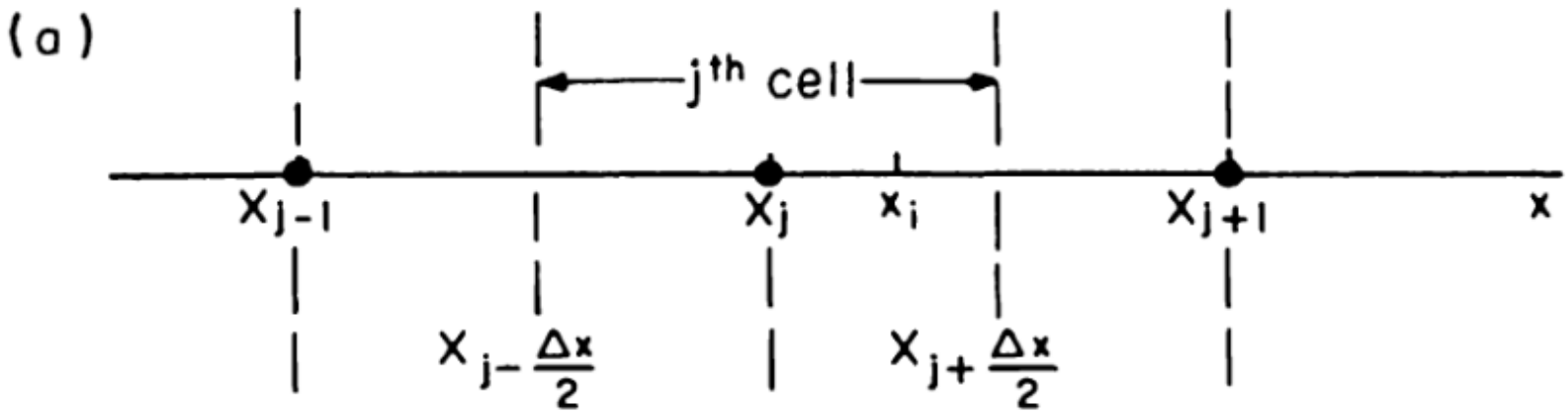
$$\boxed{\begin{aligned} v^- &= v_k + \left(\frac{q}{m}\right) \cdot E_k \cdot \left(\frac{\Delta t}{2}\right) \\ \frac{v^+ - v^-}{\Delta t} &= \frac{q}{2m}(v^+ + v^-) \times B_k \\ v_{k+1} &= v^+ + \left(\frac{q}{m}\right) \cdot E_k \cdot \left(\frac{\Delta t}{2}\right) \end{aligned}}$$

Staggered position and velocity grids; $x_k \equiv x(t_k)$, $v_k \equiv v(t_{k-1/2})$.

Details - Field evolution

1. Potential field from charge configuration:
   - $\rho(x) \xrightarrow[FFT]{} \tilde{\rho}(k) \xrightarrow[1/k^2]{} \tilde{\phi}(k) \xrightarrow[IFFT]{} \phi(x)$
   - $\frac{\phi(x+\Delta x)-2\phi(x)+\phi(x-\Delta x)}{(\Delta x)^2} = -\frac{\rho(x)}{\epsilon_0} \quad \rightarrow \quad \hat{D}_2\vec{\Phi} = -\frac{1}{\epsilon_0}\vec{\rho}$
2. Electric field from potential:
   - $E(x) = \frac{\phi(x+\Delta x)-\phi(x-\Delta x)}{2\Delta x} \quad \rightarrow \quad E = -\hat{D}_1\vec{\Phi}$

Details - Weighting (first-order)



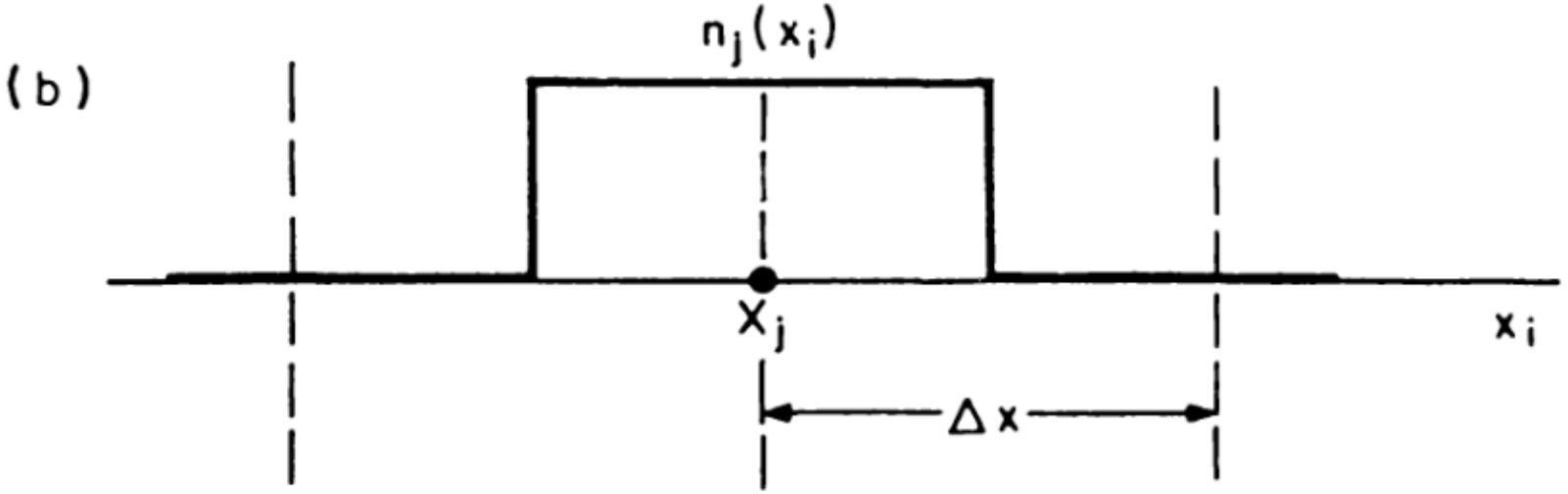
- Extrapolating charge density:

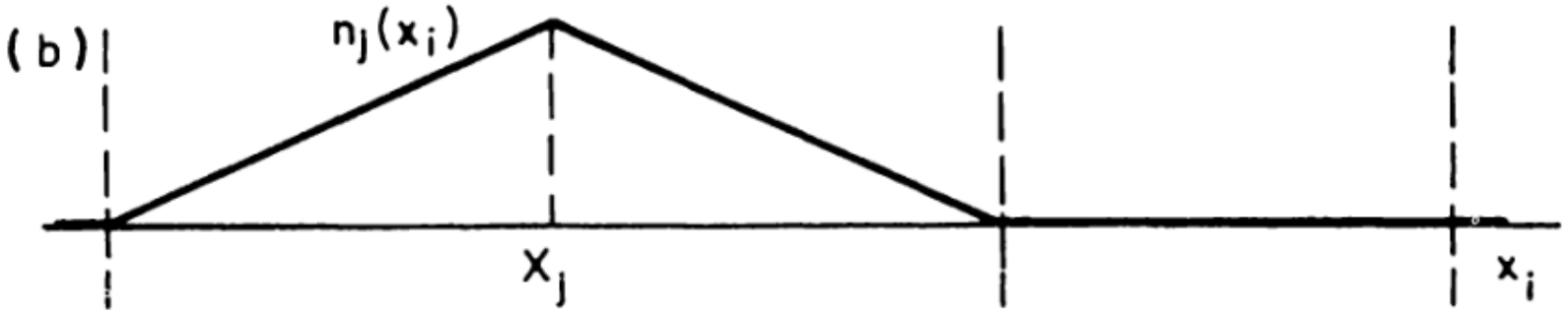
$$q_j = q_c \cdot \frac{X_{j+1} - x_i}{\Delta x} \qquad q_j = q_c \cdot \frac{x_i - X_j}{\Delta x}$$

- Interpolating electric field:

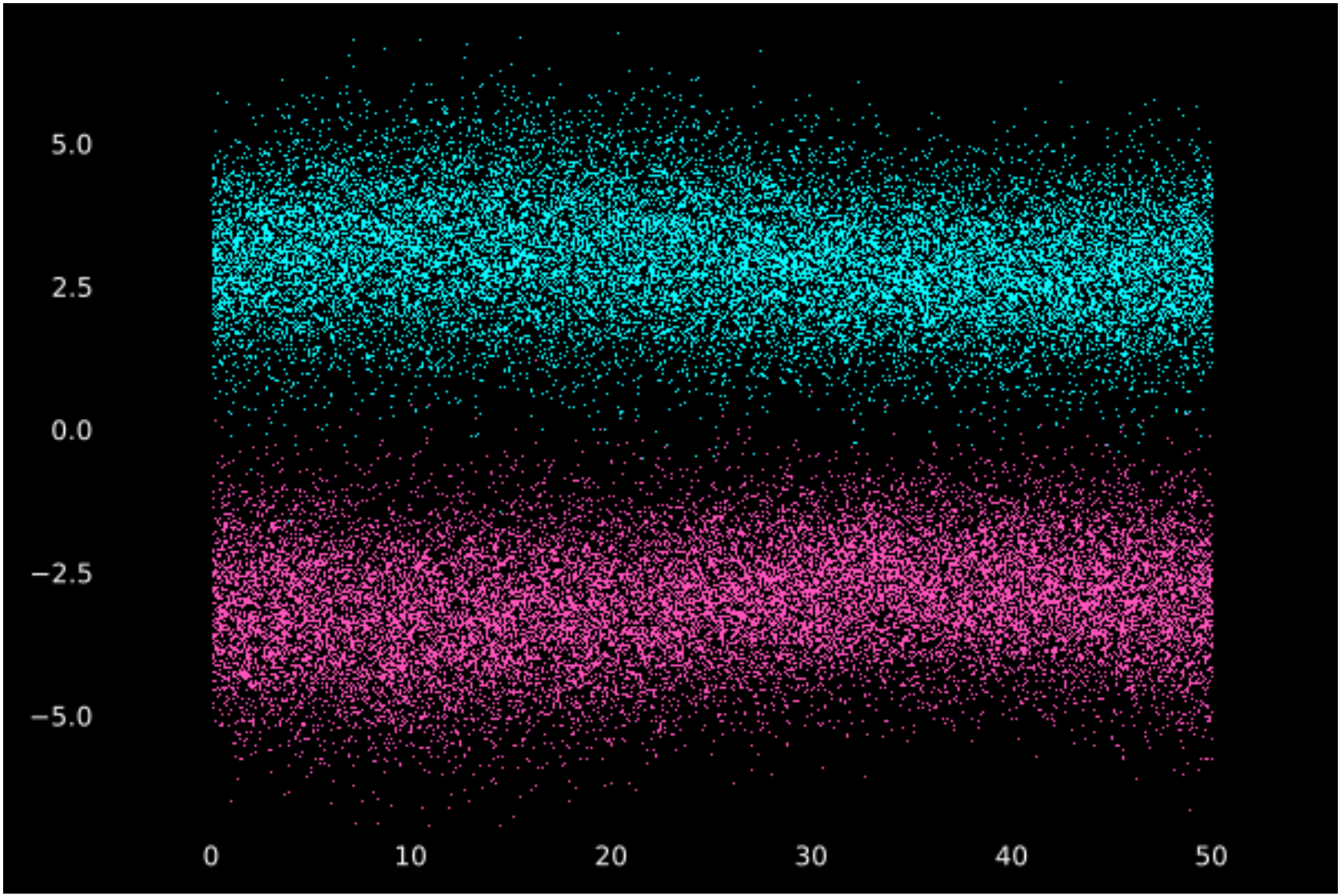
$$E(x_i) = \frac{(X_{j+1} - x_i)}{\Delta x} \cdot E_j + \frac{(x_i - X_j)}{\Delta x} \cdot E_{j+1}$$

Details - Weighting (first-order) contd.





Two-stream instability



References I

[1] American Journal of Physics 88, 159 (2020); doi: 10.1119/10.0000375

[2] Birdsall, C.K Langdon, A.B, *Plasma Physics via Computer Simulation*